Indian Statistical Institute
Mid-Semestral Examination
Topology I - MMath I

Max Marks: 40 Time: 180 minutes.

Answer all questions. You may use Theorems stated/proved in the class after correctly stating them. You may use results not discussed in the class only after proving them.

(1) Let $\mathbb{R}_d$ denote the real line with the discrete topology. Show that the dictionary order topology on the set $\mathbb{R} \times \mathbb{R}$ is the same as the product topology $\mathbb{R}_d \times \mathbb{R}$. Compare this topology with the standard topology on $\mathbb{R}^2$. [8+2]

(2) Let τ be the topology generated by the following collection of subsets of $\mathbb{R}$: (i) open intervals (a, b) and (ii) sets of the form $(a, b) - K$ where $K = \{1/n\}_{n \geq 1}$. Then,
  (a) Is $(\mathbb{R}, \tau)$ connected?
  (b) Is $[0, 1]$ compact as a subspace of $(\mathbb{R}, \tau)$?
  (c) Is $(\mathbb{R}, \tau)$ path connected?

[3+3+4]

(3) Let X be a metric space with metric d. Show that $d : X \times X \longrightarrow \mathbb{R}$ is continuous. Let X' denote a space having the same underlying set as X. If $d : X' \times X' \longrightarrow \mathbb{R}$ is continuous, then show that the topology of X' is finer than the topology of X. [5+5]

(4) Let $\pi_1 : \mathbb{R} \times \mathbb{R} \longrightarrow \mathbb{R}$ be projection to the first factor. Let A be the subspace of $\mathbb{R} \times \mathbb{R}$ consisting of points (x, y) for which either $x \geq 0$ or $y = 0$, or both. Let $q : A \longrightarrow \mathbb{R}$ be the restriction of π_1. Show that q is a quotient map. Is q an open map? Is q a closed map? [3+1+1]

(5) Show that there does not exist a continuous bijective map $f : S^1 \longrightarrow S^2$. [5]